High-Energy Inelastic Scattering of Nucleons; The Distorted-Wave Impulse Approximation*

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The effects of distorted waves on the high-energy inelastic scattering of protons by ^{12}C were investigated using the distorted-wave-impulse approximation. It was found that except for electric monopole transitions, the use of distorted waves results in a reduction of the peak differential cross section by a factor of 2 or 3, with little effect on the location or shape of the curve. It was also found that the presence of spin-orbit coupling in the distorting potential has a non-negligible effect on the proton polarization at some angles. The 2^+ level of ^{12}C at 4.43 MeV was treated in some detail in *L-S* and *j-j* coupling extremes to look at the relative effects of distortions on nonspin-flip and spin-flip matrix elements.

I. INTRODUCTION

THE high-energy scattering of protons and electrons on light nuclei has been of interest in recent years as a means of investigating nuclear structure.[1] High-energy projectiles with wavelengths on the order of nuclear dimensions or smaller are necessary, if any more than gross properties are to be observed. Nucleons are desirable as projectiles because they "see" neutrons and protons equally well, and because the strong spin and isospin coupling which characterizes the nucleon-nucleon interaction can conceivably provide valuable information about the way in which nucleon spins are distributed in the nucleus.

The outstanding drawback to use of nucleons as a tool for structure investigation is our lack of knowledge about the nucleon-nucleon interaction. In order to take full advantage of the scattering information pertaining to a given projectile interacting with a composite system, one needs to know the potential characterizing the interaction between the projectile and the constituent particles of the target, and of course we do not possess this information, if indeed a potential exists, for the two-nucleon system.

An attempt has been made[2] to circumvent this difficulty and express the nucleon-nucleus interaction in terms of the free two-nucleon transition amplitude which has been studied extensively and is experimentally accessible. This procedure depends on the use of the impulse approximation and is only expected to be valid at high energies ($\sim$100 MeV or higher). It yields expressions for the potential which produces the elastic scattering (the optical potential) and expressions for inelastic scattering amplitudes, all in terms of the free two-nucleon transition matrix. The optical potential thus obtained has received considerable attention but as yet cannot accurately predict the elastic scattering. A comparison of the predictions of calculated and empirically determined optical parameters has been made based on 180-MeV proton scattering by Johannson *et al.*,[3] illustrating the failure of the calculated parameters to give quantitative agreement with the data.

In view of the fact that elastic scattering is essentially a coherent, many-particle "transition" at any energy, this failure is not so surprising. Inelastic scattering, however, is a few-particle transition, and one may hope that it will be more readily explained by the formalism of KMT.[2] Indeed, a considerable amount of qualitative success has been obtained already, especially in the prediction of the polarization produced by normal parity $[\Delta\pi=(-1)^J]$ transitions. These calculations have been performed either in the Born approximation, where distortion effects due to elastic scattering and refraction, and absorption due to all other modes of excitation, are ignored; or in the WKB approximation[4] which takes into account some, but not all, of the optical effects.

The distortion effects are not negligible in any sense and must be taken into account. For example, at 150 MeV the cross section for inelastic transitions in ^{12}C are reduced by as much as 50% when distortions are introduced, and this attenuation persists even at very high energies.[5] In addition, the elastic scattering produces large polarizations, which indicates a spin-orbit

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[1] See, for example, A. B. Clegg, *Proceedings of the 1962 International Conference on High Energy Physics and Nuclear Structure at* CERN, edited by J. Prentki (CERN, Geneva, 1962); E. A. Sanderson, Nucl. Phys. **35**, 557 (1962); P. D. Kunz, Phys. Rev. **128**, 1343 (1962); R. S. Willey, Phys. Letters **6**, 336 (1963).

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coupling term in the optical potential. This presumably will be reflected in the inelastic polarization.

The importance for isolating the effects of distortions on the magnitude of the peak cross section arises in the following way. Transitions to low-lying states of light, even-even nuclei are thought to be collective in nature, and as such will be enhanced relative to the corresponding single-particle transitions. The amount of enhancement is critical to a description of the structure of the states participating in the transition, but is not directly observable because of the presence of distortions effects described in the previous paragraph. Therefore, in order to test the accuracy of wave functions obtained from a given structure calculation relative to the predicted collective effects, one must investigate the importance of distortions rather carefully.

II. FORMULATION

A. The Inelastic Transition Amplitude

The transition amplitude which describes the inelastic scattering of a nucleon by a nucleus can be written in the distorted-wave Born approximation (DWBA) as[1]:

$$T_{f0}=\sum_{n_a n_b}\int \chi_{m_b n_b}{}^{(-)*}(\mathbf{k},\mathbf{x})u^*(n_b)\langle\Phi_f(\mathbf{x}_j)|\times\sum_{j=1}^{A} V(\mathbf{x},\mathbf{x}_j)|\Phi_0(\mathbf{x}_j)\rangle u(n_a)\chi_{n_a m_a}{}^{(+)}(\mathbf{k}_0,\mathbf{X})d\mathbf{r} \quad (1)$$

where $\mathbf{x}$ stands for the space, spin, and isospin coordinates of the incident nucleon, $\mathbf{r}$, $\mathbf{s}$, and $\boldsymbol{\tau}$, while $\mathbf{x}_j$ represents the coordinates of the jth target nucleon. (In fact, $\mathbf{r}$ is the relative coordinate in the nucleon-nucleus center-of-mass system.)

The functions $u(n_a)$ and $u(n_b)$ are spin functions of the incident nucleon. The quantities $\chi^{(+)}$ and $\chi^{(-)}$ are distorted waves (see Appendix A) which include the effects of elastic scattering, refraction and other inelastic channels (absorption) in the incoming and outgoing channels, respectively. These latter functions are matrices in the spin space of the incident nucleon, allowing for the possibility of spin-flip caused by the spin-orbit portion of the optical potential. (This spin-flip, of course, is independent of that which may be caused by the interaction V which produces the inelastic transition.)

Equation (1) describes an event wherein the incident nucleon is scattered from an initial momentum state $\mathbf{k}_0$ and spin state m_a to a final momentum state $\mathbf{k}$, spin state m_b. The nucleus struck is excited from its ground state J_0, M_0, T_0, T_{z0}, π_0 to a given excited state J_f, M_f, T_f, T_{zf}, π_f.

B. The Effective Interaction

The quantity $V(\mathbf{x},\mathbf{x}_j)$ is the effective nucleon-nucleon interaction which produces the inelastic scattering. At low energies this is not simply related to the real interaction between two nucleons. At low energies where the mean free path of the incident nucleon is not large compared to nuclear size, multiple collisions become likely. Since, by the form of (1), multiple-step contributions to the transition have been eliminated, the form of V will have to be chosen to account for them in an effective one-step process. Not only will multiple scattering become important in the sense just described but, in addition, at each step the bombarding nucleon will see several nucleons, so that the potential to be iterated is still not the two-nucleon interaction.

We can only hope to justify use of the two-nucleon interaction at energies where the mean free path of the projectile is long compared to the nuclear size, and where its wavelength is small compared to target nucleon spacing. (These conditions are related of course.) We must also assume that the target nucleon is not "distorted" from its isolated condition by virtue of its existence within nuclear matter.

These assumptions lead us to replace $V(\mathbf{x},\mathbf{x}_j)$ by $t(\mathbf{x},\mathbf{x}_j)$, where this latter quantity is the transition matrix for free two-nucleon scattering. $t(\mathbf{x},\mathbf{x}_j)$ is a function of bombarding energy and momentum transfer in the two-nucleon collision (as well as spins and isospins), and these are not simply related to the corresponding quantities in the nucleon-nucleus collision because of refraction effects. If we assume that refraction is not too important, and in addition ignore the momentum of the struck nucleon, we can use

$$t(\mathbf{q},E_0)\delta(\mathbf{r}-\mathbf{r}_j)$$

in place of V, where E_0 is the energy of the bombarding nucleon in the laboratory system of coordinates, and

$$\mathbf{q}=\mathbf{k}-\mathbf{k}_0,$$

where $\mathbf{k}_0$ and $\mathbf{k}$ are the wave numbers for the relative nucleon-nucleus motion before and after the scattering. Estimates of the importance of using these "asymptotic" kinematics rather than the corresponding quantities which actually obtain within the nucleus seem to indicate that substantial errors result for forward scattering, which will be important for $\Delta l=0$ excitations, but less significant for all others.

The interaction which we shall use in (1) is then

$$V(\mathbf{x},\mathbf{x}_j)=t_j(\mathbf{q},E_0)\delta(\mathbf{r}-\mathbf{r}_j) \quad (2)$$

where, following KMT, $t_j(\mathbf{q},E_0)$ will be parametrized in the form

$$t_j=(2\hbar^2/(2\pi)^2 m_p)[A+B\boldsymbol{\sigma}\cdot\hat{n}\boldsymbol{\sigma}_j\cdot\hat{n}+C(\boldsymbol{\sigma}+\boldsymbol{\sigma}_j)\cdot\hat{n}+E\boldsymbol{\sigma}\cdot\hat{q}\boldsymbol{\sigma}_j\cdot\hat{q}+F\boldsymbol{\sigma}\cdot\hat{p}\boldsymbol{\sigma}_j\cdot\hat{p}]. \quad (3)$$

The unit vectors $\hat{p}$, $\hat{q}$, and $\hat{n}$ form an orthogonal coordinate system, defined by

$$\hat{q}=(\mathbf{k}-\mathbf{k}_0)/q,\quad \hat{n}=(\mathbf{k}_0\times\mathbf{k})/|\mathbf{k}_0\times\mathbf{k}|,\quad \hat{p}=\hat{q}\times\hat{n}. \quad (4)$$

m_p is the proton rest mass.

Manipulation of the matrix elements involved will be simplified if (3) is written as

$$t_j=(2\hbar^2/(2\pi)^2m_p)\sum_{SS'\lambda\lambda'} d_{\lambda\lambda'}{}^{SS'}(E_0,\mathbf{q})\sigma_\lambda{}^S\sigma_{\lambda'}{}^{S'}(j)\,, \quad (5)$$

where $\sigma_\lambda{}^S$, $\sigma_{\lambda'}{}^{S'}(j)$ are spherical tensors representing the spins of the incident and target nucleons. The coefficients $d_{\lambda\lambda'}{}^{SS'}$ in (5) are expressible in terms of the A's, B's, etc. in (3), as well as the nucleon-nucleus scattering angle θ given by

$$\theta=\cos^{-1}(\mathbf{k}_0\cdot\mathbf{k}/k_0k)\,. \quad (6)$$

For $E_0\sim 150$ MeV or greater and excitation of low-lying nuclear states, we will set $k=k_0$ for convenience. The form of the d's will be determined by the choice of axes.

We shall be interested for the moment in excitation of the collective levels of light, even-even nuclei where the transitions will be from $T=0$ ground states to $T=0$ or $T=1$ excited states. In that case, the isospin dependence of the two-nucleon scattering matrix can be handled as by KMT in a very simple way. For that reason the explicit isospin dependence of t_j in Eqs. (3) and (5) has been omitted. The A's, B's, etc. will have one set of values for $\Delta T=0$ transitions, and another for $\Delta T=1$.

C. The Matrix Element

The matrix element (1) can be calculated using (2) and (5) in a form which will facilitate use of the distorted-wave code, JULIE. This quantity is more logically written as

$$T_{f0}=\sum_{n_an_b}\int\chi_{m_bn_b}{}^{(-)*}(\mathbf{r})\langle f|\sum_j u^*(n_b)t_j \times u(n_a)\delta(\mathbf{r}-\mathbf{r}_j)|0\rangle\chi_{n_am_a}{}^{(+)}(\mathbf{r})d\mathbf{r}\,, \quad (7)$$

where the nuclear matrix element (in bra-ket notation) must be expressed in an appropriate form for the computer code.

The operator in the nuclear matrix element can be written as

$$\frac{2\hbar^2}{(2\pi)^2m_p}\sum_{J'jlSS'm'\lambda\lambda'}\frac{\delta(r-r_j)}{r_j{}^2}Y_l{}^{m'*}(\hat{r})C(lS'J:m',\lambda',M') \times\mathcal{Y}_{J'}{}^{M'}(\hat{x}_j)\langle n_b|\sigma_\lambda{}^S|n_a\rangle d_{\lambda\lambda'}{}^{SS'}\,, \quad (8)$$

where $C(lS'J:m',\lambda',M')$ is a Clebsch-Gordan coefficient, Y_l is a spherical harmonic, and $\mathcal{Y}_{J'}$ is a spherical tensor of rank J' acting on the coordinates of the $\mathcal{J}$th nucleon and expressed by

$$\mathcal{Y}_{J'}{}^{M'}(\hat{x}_j)=\sum_{m''}C(lS'J':m'',\lambda',M')Y_l{}^{m''}(\hat{r}_j)\sigma_{\lambda'}{}^{S'}(j)\,. \quad (9)$$

Equation (7) can now be written as

$$T_{f0}=\frac{2\hbar^2}{(2\pi)^2m_p}\sum_{SS'\lambda\lambda'}d_{\lambda\lambda'}{}^{SS'}\sum_{J'lm'}C(lS'J:m',\lambda',M') \times\sum_{n_an_b}\langle n_b|\sigma_\lambda{}^S|n_a\rangle\int\chi_{m_bn_b}{}^{(-)*}(\mathbf{r})Y_l{}^{m'*}(\hat{r}) \times\langle f|\sum_{j=1}^{A}\frac{\delta(r-r_j)}{r_j{}^2}\mathcal{Y}_{J'}{}^{M'}(\hat{x}_j)|0\rangle\chi_{n_am_a}{}^{(+)}(\mathbf{r})d\mathbf{r}\,. \quad (10)$$

D. Nuclear Wave Functions

The calculation of T_{f0} in (10) requires determination of a nuclear matrix element of the form

$$\langle\Phi_f(\mathbf{x}_1,\cdots\mathbf{x}_A)|\sum_{j=1}^{A}\mathcal{O}(\mathbf{x},\mathbf{x}_j)|\Phi_0(\mathbf{x}_1,\cdots\mathbf{x}_A)\rangle\,, \quad (11)$$

where Φ_0 and Φ_f are antisymmetrized wave functions. In particular, we wish to consider collective excitations of even-even nuclei which have 0^+, $T=0$ ground states and final states with angular-momentum states J, M, even or odd parity and $T=0$ or 1.

We shall restrict our attention for the moment to hole-particle wave functions wherein the excited states of such nuclei are described by the coupling of hole-particle pairs to the quantum numbers of the final state. The single-particle motions are described by harmonic oscillator functions labeled by n, l, and $j(=l\pm\frac{1}{2})$; and all hole-particle pairs which are allowed by the selection rules of the transition and which correspond to one-quantum transitions are included. The expression for (11) in the case of an extreme j-j coupled, single-particle transition will now be developed: extension to the more general wave functions will then be trivial.

1. Single-Particle Excitations

We shall represent the ground state of the nucleus as any number of closed shells which do not participate in the excitation, plus an even number of nucleons in an outermost shell, j-j coupled to $J=0$. Let n_A, l_A and j_A, m_A be the quantum numbers of a nucleon in this shell, and let n_B, l_B, j_B, and m_B designate the quantum numbers of the nucleon excited to a higher shell and coupled to the remaining core to produce the angular momentum of the excited state.

Utilizing the language of second quantization, let $a_m{}^\dagger$ and a_m, respectively, create and destroy particles in the A shell with magnetic quantum number m, while $b_m{}^\dagger$ and b_m are analogous operators in the B shell. If we represent the ground state of the nucleus by $|0\rangle$, the the excited state for a single-particle excitation can be written as

$$|J_f,M_f\rangle=\sum_{m\mu}C(j_Bj_AJ_f:\mu,m,M_f) \times(-1)^{j_A+m}a_{-m}b_\mu{}^\dagger|0\rangle\,, \quad (12)$$

where we have used the fact that the state

$$(-1)^{j_A+m}a_{-m}|0\rangle$$

has the rotational properties of a particle of angular momentum j_A and magnetic quantum number m. The particle-hole pair in (12) has been coupled to J_f, M_f.

The operator in Eq. (11) can be written as

$$\sum_{\alpha,\beta} n_\alpha{}^\dagger n_\beta \langle\alpha|\mathcal{O}|\beta\rangle\,, \tag{13}$$

where $\langle\alpha|\mathcal{O}|\beta\rangle$ is the matrix element of $\mathcal{O}(\mathbf{x},\mathbf{x}_j)$ between single-particle states. The symbols n_α and n_β each stand for sets of j and m. Designating the quantity in (11) as $\langle J_fM_f|\mathcal{O}|00\rangle$, and using (12) and (13), we obtain

$$\langle J_fM_f|\mathcal{O}|00\rangle=\sum_{m\alpha\beta\mu} C(j_Bj_AJ_f\colon \mu mM_f)(-1)^{j_A+m} \times\langle\alpha|\mathcal{O}|\beta\rangle\langle 0|a_{-m}{}^\dagger b_\mu n_\alpha{}^\dagger n_\beta|0\rangle\,. \tag{14}$$

Using Wick's theorem, or working directly with the commutation rules, we arrive at

$$\langle J_fM_f|\mathcal{O}|00\rangle=\sum_{m\mu} C(j_Bj_AJ_f\colon \mu mM_f) \times(-1)^{j_A+m}\langle j_B,\mu|\mathcal{O}|j_A,-m\rangle\,. \tag{15}$$

With this result, the nuclear matrix element in (10) becomes

$$\sum_{m\mu} C(j_Bj_AJ_f\colon \mu mM_f)(-1)^{j_A+m} \times\langle j_B,\mu|\frac{\delta(r-r_j)}{r_j{}^2}\mathcal{Y}_{J'}{}^{M'}(\hat{x}_j)|j_A,-m\rangle\,, \tag{16}$$

where r_j is the coordinate in the bra and ket. Equation (16) reduces at once to

$$\sum_{m\mu}(-1)^{j_A+m}C(j_Bj_AJ_f\colon \mu mM_f)u_B{}^*(r) \times u_A(r)\langle j_B,\mu|\mathcal{Y}_{J'}{}^{M'}|j_A,-m\rangle\,. \tag{17}$$

Equation (17) was obtained by reasoning about one type of particle. Since for this problem neutrons and protons contribute equally, the matrix element in (17) should be multiplied by $\sqrt{2}$ (which gives a factor of two in the cross-section). Note that for p-n scattering this factor would be absent.

Applying the Wigner-Eckart theorem to (17) in the form

$$\langle j'm'|\omega_k{}^q|jm\rangle=(-1)^{2k}C(jkj'\colon mqm')\langle j'\|\omega_k\|j\rangle\,, \tag{18}$$

we obtain for the nuclear matrix element

$$\sqrt{2}u_B{}^*(r)u_A(r)\langle j_B\|\mathcal{Y}_{J'}\|j_A\rangle\sum_{m\mu}(-1)^{j_A+m} \times C(j_Bj_AJ_f\colon \mu mM_f)C(j_AJ'j_B\colon -mM'\mu)\,. \tag{19}$$

The sum on magnetic quantum numbers can be performed at once, and yields at last

$$\sqrt{2}u_B{}^*(r)u_A(r)(\hat{j}_B/\hat{J}_f)\delta_{J'J_f}\delta_{M'M_f}\,, \tag{20}$$

where $\hat{j}$ is short-hand for $(2j+1)^{1/2}$.

Substituting (20) into (10), we arrive at an expression for the transition matrix element in the case of a single-particle excitation in the j-j coupling limit:

$$T_{f0}=\frac{2\sqrt{2}\hbar^2}{(2\pi)^2m_p}\frac{\hat{j}_B}{\hat{J}_f}\sum_{lSS'\lambda\lambda'} d_{\lambda\lambda'}{}^{SS'}C(lS'J_f\colon M_f-\lambda',\lambda') \times\langle\tfrac{1}{2}\|\sigma^S\|\tfrac{1}{2}\rangle\langle j_B\|\mathcal{Y}_{J_f}\|j_A\rangle \times\sum_{n_an_b} C(\tfrac{1}{2}S\tfrac{1}{2}\colon n_a\lambda n_b)\int\chi_{m_bn_b}{}^{(-)*}(\mathbf{r})Y_l{}^{M_f-\lambda'*}(\hat{r}) \times u_B{}^*(r)u_A(r)\chi_{n_am_a}{}^{(+)}(\mathbf{r})d\mathbf{r}\,. \tag{21}$$

If the more general hole-particle wave functions are used, the resulting transition matrix T_G will be a sum of terms like (21) in the form

$$T_G=\textstyle\sum_\nu \alpha_\nu T_{f0}(\nu)\,, \tag{22}$$

where to each value of ν in the sum, there corresponds a set n_A, j_A, l_A, n_B, j_B, and l_B, and the coefficients α_ν are obtained from structure calculations.

Equation (22) represents the quantity we wish to compute; with it we obtain a description of the inelastic scattering to within the errors introduced by our assumptions. Before calculating (22), however, we shall attempt to isolate to some extent the contributions to the form of the transition amplitude expected from the effects of distorted waves.

These effects will be considered in terms of the extreme single-particle model for the nuclear wave functions which will eliminate all save the gross effects of nuclear structure. We shall look at the scattering at bombarding energies of 90, 156, and 310 MeV, since the two-nucleon coefficients are readily available at these energies. It should be kept in mind that the approximations made here become suspect at, or in the neighborhood of, 100 MeV, so that some care must be exercised when viewing the results obtained at 90 MeV.

III. DISTORTED WAVE CALCULATIONS

A. Optical Parameters

The parameters which specify the distorting optical potential described in Appendix A can be obtained from the analysis of elastic proton scattering data. Such an analysis has been performed at 180 MeV by Johannson *et al.*,[3] and we shall rely on the optical parameters quoted by these authors which are listed in Table I.

The two-nucleon transition matrix is available at 90, 156, and 310 MeV, while we only have the parameters for the optical potential at 180 MeV. Although it might seem that interpolation of the t matrix to 180 MeV is indicated, relatively more theoretical information on the energy dependence of the strengths of the optical potential is at hand, which relates the central and spin-orbit portions of the optical potential to the central and spin-orbit portions of the two-nucleon scattering

TABLE I. The optical parameters at 180 MeV were obtained from Ref. 3. The potential strengths at the other three energies were obtained by extrapolation from 180 MeV.

	180 MeV	90 MeV	156 MeV	310 MeV
V	16.	22.6	18.2	5.32
W	10.	11.1	10.2	12.1
r_c	1.26	1.26	1.26	1.26
r	1.	1.	1.	1.
a	0.5	0.5	0.5	0.5
V_S	2.5	2.88	2.62	2.1
W_S	$-1.$	-1.53	-1.07	-0.806
r'	1.34	1.34	1.34	1.34
a'	0.5	0.5	0.5	0.5

amplitude. Using these relations, the 180-MeV potential strengths were extrapolated to 90, 156, and 310 MeV. The well radii and diffuseness were not varied. The optical potentials thus obtained are displayed in Table I.

B. 2^+ Level of ^{12}C

We consider the transition from the 0^+, $T=0$ ground state of ^{12}C to the 2^+, $T=0$ level at 4.43 MeV. This corresponds to $l=J_f=2$ in Eq. (21). This is a $1p-1p$ particle transition: from $j_A=\frac{3}{2}$ to $j_B=\frac{1}{2}$ in j-j coupling, and $\Delta L=2$, $\Delta S=0$ in L-S coupling. The initial and final radial wave functions are $1p$ functions. If we use harmonic oscillator functions,

$$u_{1p}(r)=(2^{11/2}\alpha^{5/2}/3\sqrt{\pi})^{1/2}re^{-\alpha r^2}.$$

The parameter α should be related to the charge radius which is inferred from elastic electron scattering. We use the value $\alpha=0.188$ F^{-2}.

Oscillator functions probably yield a reasonable radial distribution for small r, but fail conspicuously for large r because of their Gaussian tail where an exponential tail is required. In the plane-wave limit where distortion effects are ignored, the inelastic cross section is proportional to

$$\left|\int u_B^*(r)j_L(qr)u_A(r)r^2dr\right|^2, \tag{23}$$

where q is the magnitude of the momentum transfer. L is the orbital angular momentum transfer in the scattering. Because the Bessel function damps out rapidly for increasing argument, one can say to zeroth order that for large q only small r is important. Hence, for high-energy scattering (and not too close to the forward direction), harmonic oscillator functions will probably be adequate. However, when the large r portion of the wave functions becomes important relative to the small r portion, oscillator functions may prove inadequate because of their Gaussian dependence in this region. This tail region becomes important for forward scattering: for $L=0$ where the inelastic cross section peaks in the forward direction, the effects should be particularly large. In the DWBA the cross section is proportional, not to (24), but to an average over various momentum transfers produced by refraction. Thus, although the "asymptotic moment transfer" $\mathbf{q}=\mathbf{k}-\mathbf{k}_0$ might correspond to a case in the BA where the tail region is unimportant, refraction effects could produce a nonnegligible contribution from the large r part of the wave function. A more important effect in the DWBA is produced by absorption due to the imaginary part of the optical potential. This produces an attenuation of the wave function within the nuclear volume and thus emphasizes the tail of the wave function.

Keeping these remarks in mind, we shall nevertheless use oscillator functions for the nuclear radial wave function. These will provide form factors of sufficient accuracy for explorative purpose, although they will be replaced by solutions of a Saxon-Wood potential in later detailed calculations. It should be pointed out that the polarization results may be less sensitive to the radial wave functions than the cross section. In the plane-wave limit, the polarization is independent of the radial integral. For L-S coupling, this is also approximately true in the DWBA, and hence we may expect the form of the radial functions to be relatively unimportant in calculating the polarization.

C. Zero-Range, Scalar Interaction

The effects of distorted waves on the inelastic scattering cross section and polarization are most clearly seen if we put $t(\mathbf{q},E_0)=1$ in Eq. (2), corresponding to a zero-range, scalar, two-nuclear interaction. The transition amplitude in this case is given by

$$T_{f0}=\frac{2\sqrt{2}\hbar^2}{(2\pi)^2m_p}\frac{\hat{j}_B}{\hat{J}_f}\langle j_B\|\mathcal{Y}_{J_f}\|j_A\rangle\sum_{n_an_b}\int\chi_{m_bn_b}^{(-)*}(\mathbf{r})\times Y_{J_f}^{M_f*}(\hat{r})u_B^*(r)u_A(r)\chi_{n_am_a}^{(+)}(\mathbf{r})d\mathbf{r}. \tag{24}$$

Only the lowest permitted l transfer in any transition has been retained in (24). The reduced matrix element contains the coupling information for the initial and final states involved. This transition amplitude will be calculated exactly, using previously developed computer codes.

The importance of distortions on the inelastic transition can be expected to vary with energy, not only because of the changing wavelength of the incident particle but also because of the energy variation of the parameters which specify the optical potential which produces the distortions. The contributions to the distortion effects from the various portions of the optical potential were obtained at the three energies by setting the appropriate strengths to zero. To facilitate discussion, the abbreviations in Table II have been adopted. V and W, and V_s and W_s are the real and imaginary strengths, respectively, of the central and spin-orbit portions of the optical potential, while Z is the nuclear charge. The cross sections and polarizations obtained

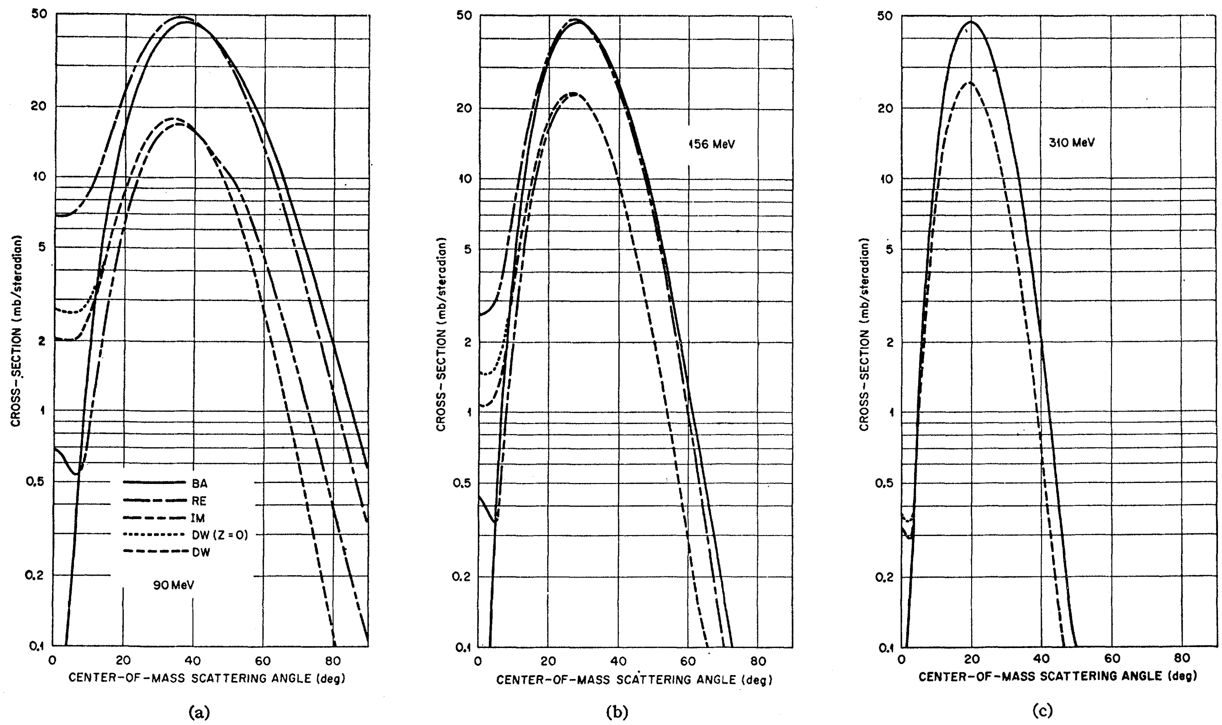


FIG. 1. Effects of distorted waves on the differential cross section using a zero-range, scalar, two-nucleon force. The curve labeling is explained in the text and in Table II. The normalization has been chosen for convenience.

are shown in Figs. 1 and 2. The curves labeled INI and FIN on the polarization plots resulted from including the spin-orbit coupling only in the initial channel and only in the final channel, respectively. The curve labeled DW contains the spin-orbit coupling in both channels.

D. Two-Nucleon Interaction

The cross sections and polarizations were then calculated using the full two-nucleon transition matrix including its momentum transfer and spin dependence. This was done for extreme j-j and extreme L-S coupling. The results obtained are shown in Figs. 3–5.

IV. RESULTS

A. Zero-Range, Scalar Force

The inelastic cross sections shown in Figs. 1–3 for a zero-range, scalar, nucleon-nucleon interaction clearly

TABLE II. The labels defined here indicate which of the optical potential strengths are zero in the distorted wave calculations. The notation $\neq 0$ indicates that the corresponding strength takes the appropriate value from Table I.

	V	W	V_s	W_s	Z
BA	0	0	0	0	0
RE	$\neq 0$	0	0	0	0
IM	0	$\neq 0$	0	0	0
DW ($z=0$)	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	0
DW	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$

show the effects of distorted waves compared to the plane wave results, as well as the relative importance of distortions produced by the real part of the optical potential (reflection and refraction) and the imaginary part (absorption). The following features are significant. (These remarks are quite accurate for the 156- and 310-MeV results and are approximately true at 90 MeV.)

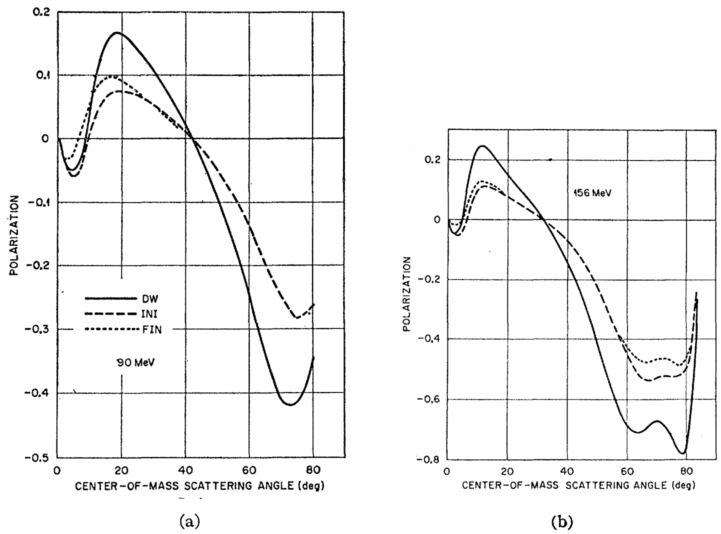
1. Refraction and reflection due to Coulomb scattering and the real part of the optical potential are almost negligible except for forward scattering.

2. The peak cross section occurs at essentially the same angle with and without distortions.

3. The over-all effect of distorted waves is a reduction of the peak plane-wave inelastic cross section by a factor of about 2. These features are consistent with the sults obtained by Kawai *et al.*[6] for the 1^+ level of ^{12}C at 15.11 MeV (which is an $l=0$, spin-flip transition).

The experimentally observable features of the cross sections at these energies are the magnitude of the peak cross section and its angular position. Details of shape are somewhat uncertain. The results quoted here indicate that the effects of distorted waves relevant to reproducing experimentally observed cross sections are well represented if the central, absorptive portion of the optical potential is accurately determined. This requires the evaluation of three parameters at each energy: the radius of the well, the diffuseness, and the potential strength. If the first two parameters are de-

[6] M. Kawai, T. Terasawa, and K. Izumo (to be published).



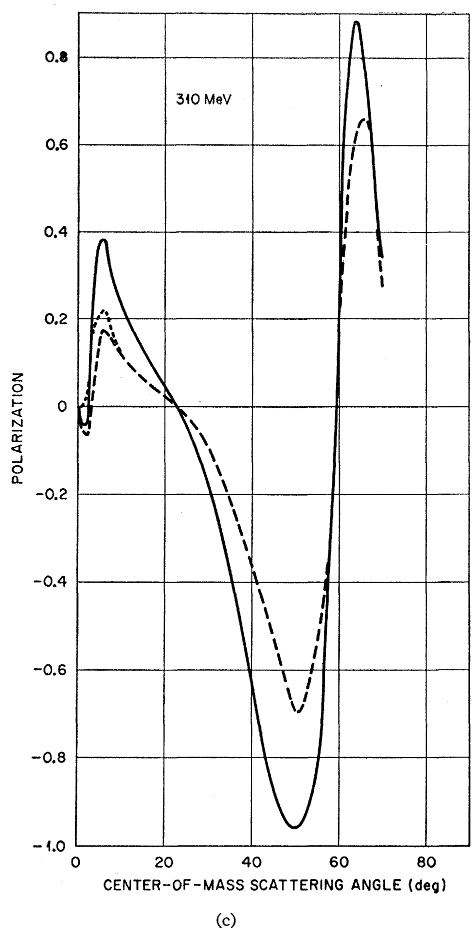


FIG. 2. Polarization produced by the spin-orbit portion of the optical potential present only in the initial channel (INI), only in the final channel (FIN), and in both channels (DW).

termined at some energy by a complete investigation of the elastic scattering data and then regarded as essentially the same at other energies, then the potential strength is determined by the total reaction cross section.

The polarizations produced by the spin-orbit coupling in the optical potential are shown in Fig. 2. In the neighborhood of the peak cross section, the polarizations produced with the spin-orbit force present only in the initial channel or only in the final channel are essentially equal, and seem to add to produce the polarization that is produced with the spin-orbit force present in both channels simultaneously; that is, the spin-orbit is essentially perturbative in the region of interest. This fact has been dealt with previously.[7]

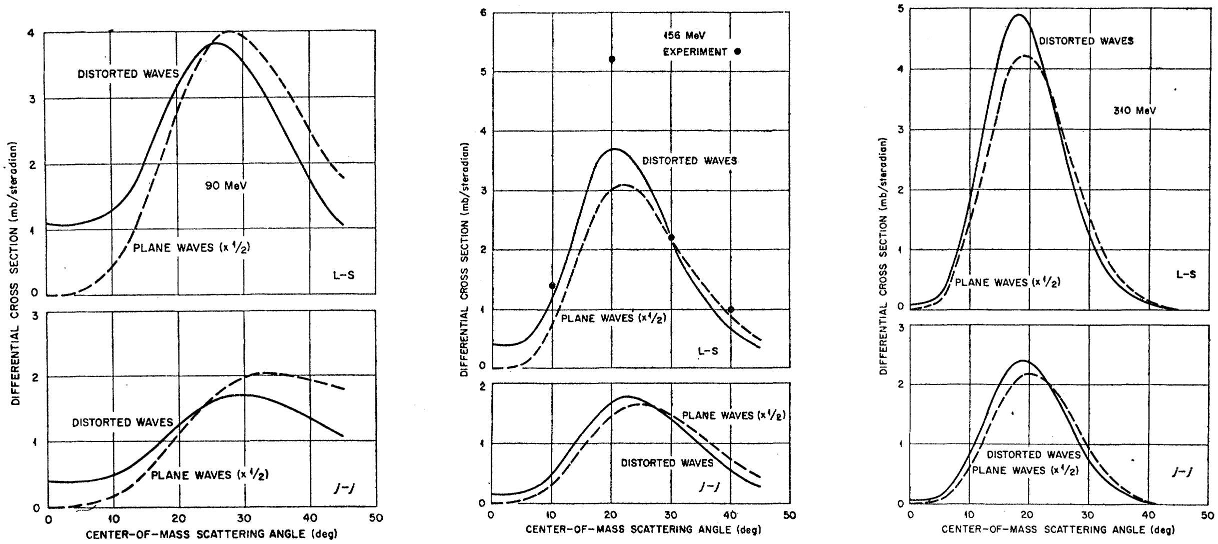
The discussion in this section has indicated that the reflection and refraction effects produced by the real part of the optical potential are essentially unimportant at these energies. Strictly, we have only established that reflection can be ignored, since the use of a force with a finite range could make refraction of considerably importance. Calculations including finite-range effects are under way and will be reported.

B. Two-Nucleon Interaction

The transition considered here is probably not well represented by either coupling extreme which we have used here. We can immediately rule out pure j-j coupling from our results as compared to experiment in Fig. 4(b). The 156 MeV polarization data clearly favors the L-S coupling extreme. In addition, the collective enhancement expected should, in this case, multiply the results obtained here by a factor of 2, which increases the L-S result at 156 MeV up close to the experimental curve, but leaves the j-j cross section still too small.

Concentrating on the L-S results, it can be seen that the contributions to the inelastic polarization from the optical spin-orbit coupling are not negligible at any energy. This spin-orbit force is virtually negligible in configuration space compared to the other forces present, but it is not small in spin space, and contributes substantially to the polarization predicted. The optically

[7] R. M. Haybron, H. McManus, A. Werner, R. M. Drisco, and G. R. Satchler, Phys. Rev. Letters 12, 249 (1964).



(a) (b) (c)

FIG. 3. Differential cross sections in the distored-wave impulse approximation for extreme *j-j* and extreme *L-S* coupling. Note that the plane wave results have been scaled by a factor of $\frac{1}{2}$ for convenience. The data points (black triangles) at 156 MeV are taken from the data of Jacmart *et al.* [J. C. Jacmart, J. P. Garron, M. Riou, and C. Ruhla, in *Direct Interactions and Nuclear Reaction Mechanisms* edited by E. Clemental and C. Villi (Gordon and Breach, Science Publishers, New York, 1963)], on experiments at 156 MeV.

produced polarization essentially adds to that produced by the two-nucleon force. The fact that the former contribution goes to zero near the peak two-nucleon contribution accounts for the success of the KMT, plane-wave predictions, for normal parity transitions. The KMT predictions should be expected to fail for the abnormal parity transitions, where the two-nucleon interaction produces small polarizations, and calculations on the 15.11 MeV, 1^+ level of ^{12}C indicate this to be the case.

Information about the ratio of the spin-flip to nonspin-flip matrix elements can be obtained from the curves in Fig. 3(b). The *L-S* coupled curve is proportional only to the nonspin-flip matrix element both for BA and DW. Therefore, dividing the DW results by the BA results yields a function of scattering angle which reflects the effects of distortions on the nonspin-flip matrix element. The *j-j* cross section is determined by a mixture of spin-flip and nonspin-flip, but we can still calculate a ratio such as that described above. These ratios for the 156-MeV cross sections are given in Table III.

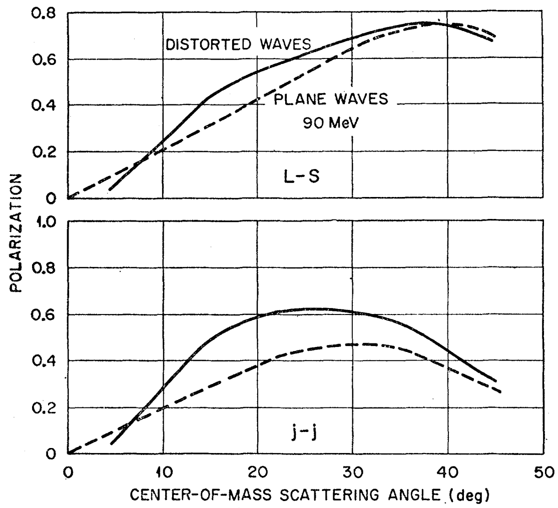
It can be seen that the ratios are almost identical. Hence, the distortion effects are essentially the same for the spin-flip and nonspin-flip matrix elements. In the language of KMT, λ is essentially a constant and has the same value as in the case of plane waves. This is merely a reflection of the fact that the spin-orbit coupling in the optical potential is small as far as its effect on the differential cross section is concerned.

The fact that λ is essentially a constant is not obvious from the polarization results because here the spin-orbit portion of the distorting potential is not small. (That is, the spin-orbit term is small in configuration space, but not small in spin space compared to the nucleon-nucleon force.) This would also be true for the angular correlation function. In both cases one must account for the optical spin-orbit effects before attempting to deduce the value of λ or its angular dependence. This point is rather significant, since, as noted by Clegg (see Ref. 1), the angular dependence of λ (or the lack of it) contains considerable information about the nuclear states involved. Analysis of the correlation function at 150 MeV[8] without the inclusion of spin-orbit effects has led to a rather substantial angular dependence for the value of λ. In view of the results obtained here, it is possible that much of this variation is due to the spin-orbit dis-

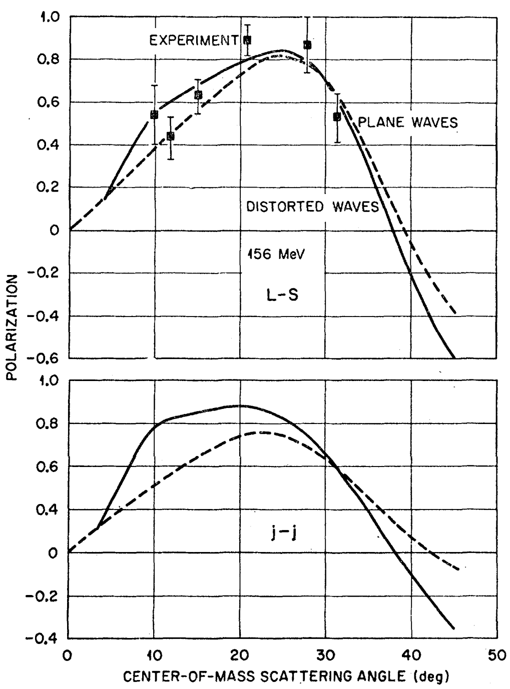
TABLE III. The first two columns are the ratio of distorted-wave cross section to plane-wave cross section for *L-S* and *j-j* coupling, respectively. λ (G.V.) is the spin-flip ratio calculated from the Gillet transition density. λ (exptl.) is the same ratio determined from p-γ correlation measurements as described in Ref. 8.

$\theta(0)$	*L-S* $d\sigma(\mathrm{DW})/d\sigma(\mathrm{BA})$	*j-j* $d\sigma(\mathrm{DW})/d\sigma(\mathrm{BA})$	λ(G.V.)	λ(exptl.)
0	...	...	0.0295	
5	3.48	3.39		
10	0.830	0.826	0.0335	
15	0.671	0.654		0.34 ±0.105
20	0.614	0.587	0.0741	
25	0.553	0.525		0.10 ±0.5
30	0.483	0.465	0.0725	
35	0.422	0.410		0.069±0.013
40	0.385	0.366	0.113	
45	0.363	0.330		

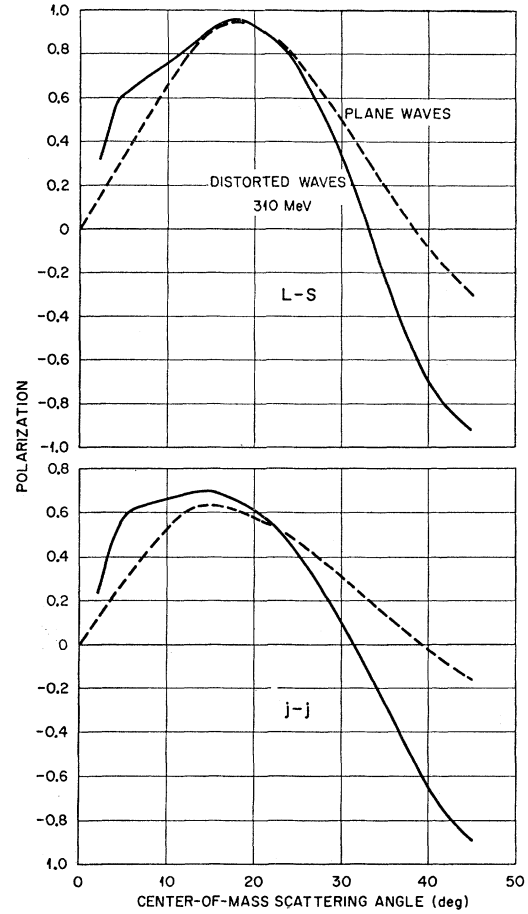
[8] G. L. Salmon *et al.*, Proc. Phys. Soc. (London) **79**, 14 (1962).



(a)



(b)



(c)

FIG. 4. Polarizations in the DWIA for both coupling extremes. The data points for the 156-MeV curves (black circles with error flags) were taken by R. Alphonce, A. Johannson, and G. Tibell [Nucl. Phys. 4, 672 (1957)] at 155 MeV. Plane-wave results are dashed.

tortions rather than effects of nuclear structure, and this point will be investigated more closely.

The Gillet wave functions[9] for the ground state and first excited state define λ for the transition under consideration. This has been calculated and tabulated in Table III, compared with the values obtained in Ref. 8. The disagreement is probably due in part to the neglect of spin-orbit effects in Ref. 9. The large discrepancy at 15°, which is just where spin-orbit distortions have a large effect, and the relatively good agreement at 25° and 35°, where the spin-orbit distortions are small, indicate that spin-orbit coupling probably must be taken into account in the analysis of p-γ correlations to determine λ values.

V. OTHER LEVELS OF ^{12}C

The effects of distorted waves depend on the transition being considered. In order to display this fact, the plane-wave and distorted-wave cross sections and polarizations for several levels of ^{12}C have been included in Fig. 5. The single-particle transition assumed to

[9] V. Gillet, thesis, Paris, 1962 (unpublished).

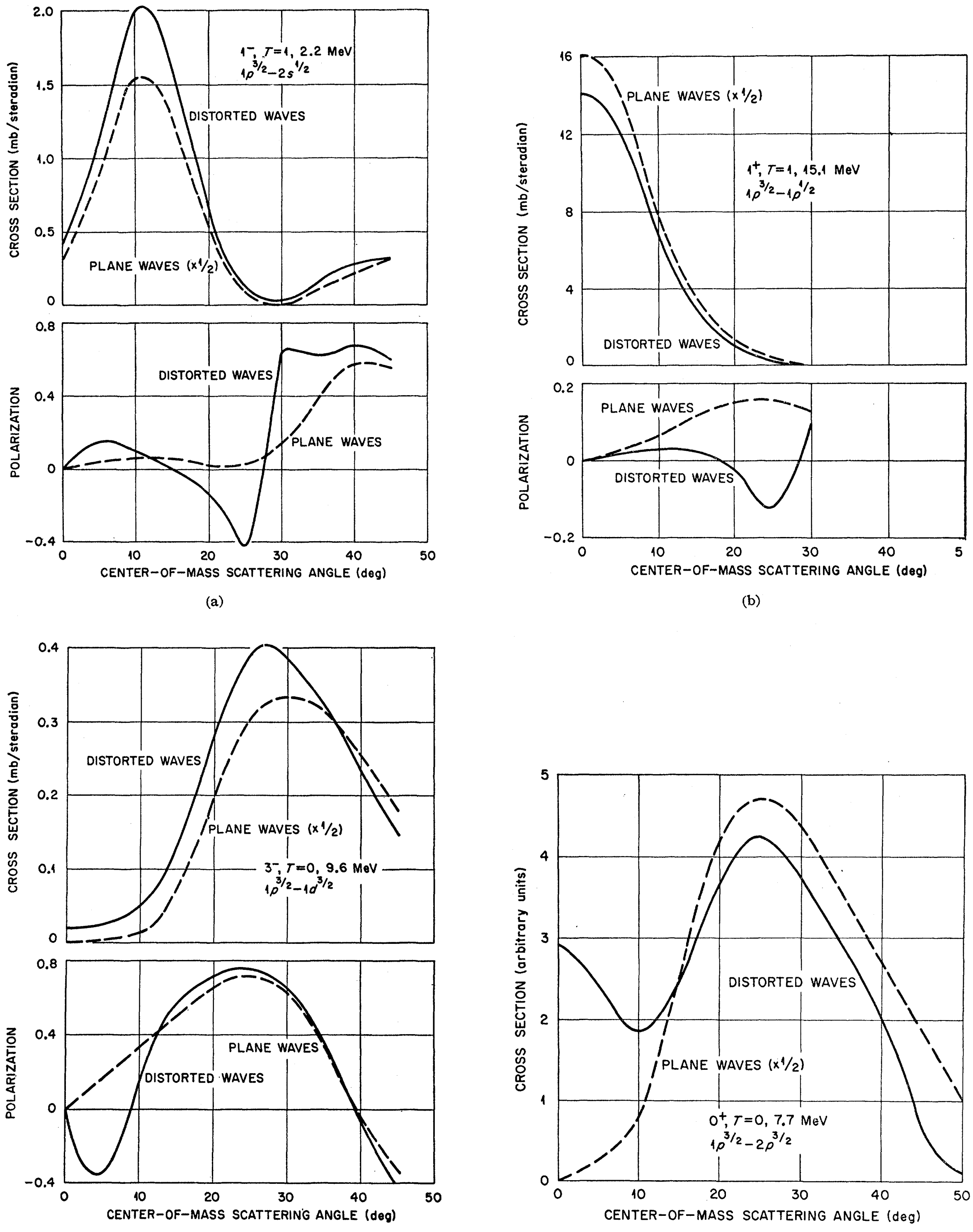


FIG. 5. Cross section and polarization for 156-MeV protons. The solid lines are distorted waves; the dashed lines are plane waves. The plane-wave cross sections have been reduced by $\frac{1}{2}$ in every case.

account for the excitation in every case labels the corresponding figure.

The general features of the DWIA cross sections for these levels can be seen to conform to the conclusions arrived at for the 2^+ level, except for the 7.7, 0^+ level in Fig. 5(d). If this transition is predominately $1p$-$2p$, the plane wave cross section is zero in the forward direction. Distorted waves, however, produce a nonzero forward cross section due to refraction. This forward peaking has been observed by Tibell.[10] Based on a WKB calculation, Brink[11] suggested that this peak is a distorted-wave effect, a conclusion supported by our result.

The polarization curves shown indicate that except for parity-favored, nonisospin-flip transitions, the plane-wave approximation does not yield reasonable results.

VI. REMARKS

We have investigated high energy, inelastic, proton scattering in order to determine the effects of distorted waves on the predicted cross sections and polarizations for transitions in light nuclei. The point of this is primarily to develop a systematic way to test the detailed structure calculations which are being done, in particular those employing the notion of hole-particle coupling to represent collective effects. This report has presented the apparatus with which we shall proceed and a few examples. It was felt that in view of the substantial improvement in the treatment of the distortion effects as compared to any previous calculations, a preliminary discussion was in order. In a forthcoming communication, we shall present cross sections and polarizations for a variety of levels in ^{12}C, ^{16}O, and ^{40}Ca, the three nuclei investigated by Gillet.

The 90-MeV curves included here would seem to be out of place in a discussion devoted to high energy, impulse approximation scattering, and indeed we should not like to express faith in these results at this point. However, there are a number of reasons why one could hope that the DWBA matrix element in Eq. (1), with an effective interaction based on the free two-nucleon scattering operator, could conceivably work at energies below the 100-MeV limit which is commonly quoted.

The scattered wave function is strongly damped inside the nucleus due to absorption. At 150 MeV, calculations indicate that contributions to the inelastic cross section fall off very rapidly inside the nucleus, not an unfamiliar result. Since the reaction cross section is known to stay constant to much lower energies, this emphasis of the surface of the nucleus should persist, as the bombarding energy is lowered. Hence, the inelastic transition is favored to occur in a region where the nuclear potential is relatively weak and where the nucleon density is low. Therefore, if the impulse approximation works well at 150 MeV, it is conceivable that it could also work at lower energies. Preliminary calculations on the excitation of the 5^- level of ^{40}Ca with 55-MeV protons by Terasawa and Satchler[12] indicate that the DWIA might be applicable there if corrected for refraction.

[10] G. Tibell (to be published).

[11] D. M. Brink (private communication).

ACKNOWLEDGMENTS

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APPENDIX A

The distorted waves $\chi_{mm'}^{(\pm)}$ satisfy the equation

$$\left[\nabla^2+k^2-\frac{2\mu}{\hbar^2}(U+U_S\mathbf{L}\cdot\boldsymbol{\sigma})-V_c\right]\chi_{mm'}^{(\pm)}(\mathbf{r})=0, \quad \text{(A1)}$$

where the choice of $+$ or $-$ represents a choice of outgoing or incoming boundary conditions. k is the wave number of the relative motion of the target and projectile, μ is the reduced mass, and V_c the Coulomb potential. The optical potentials are defined by

$$U=-V/(1+e^x)-iW/(1+e^{x'}), \quad \text{(A2)}$$

where

$$x=(r-r_0M^{1/3})/a$$

and

$$x'=(r-r_0'M^{1/3})/a'. \quad \text{(A3)}$$

M is the nuclear mass. U_S is given by

$$U_S=-2(V_S+iW_S)\frac{1}{r}\frac{d}{dr}\left(\frac{1}{1+e^x}\right). \quad \text{(A4)}$$

V_c is the Coulomb potential produced by a uniform spherical charge distribution of radius

$$R_c=r_cM^{1/3}. \quad \text{(A5)}$$

The parameters which specify the optical potential at a given energy are displayed in Table I.

[12] T. Terasawa and G. R. Satchler (private communication).